# The NKM with a supply shock

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# The New Keynesian Model with a supply shock

## ... and a shock to the $r_t^n$ that ... vanishes

$$
\begin{aligned}
IS: &\quad \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \\
AS: &\quad \pi_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} + s_t \\
MP: &\quad i_t = \pi_t + r_t^n + \phi_\pi \pi_t + \phi_y \hat{y}_t \\
\text{Shocks}: &\quad r_t^n = \rho_r \cdot r_{t-1}^n + \varepsilon_t^r \ , \qquad s_t = \rho_s \cdot s_{t-1} + \varepsilon_t^s
\end{aligned}
$$

- $\{i, r_t^n, \hat{y}, \pi, s_t, \varepsilon_t\}$: nominal interest rate, natural real interest rate, output-gap, inflation rate, supply shock, and a random disturbance.
- $\{\sigma, \kappa, \beta, \phi_\pi, \phi_y, \pi_t^*, \rho\}$ are parameters
- Forward-looking variables: $\hat{y}_t, \pi_t$
- Backward-looking variables: $r_t^n, s_t$
- Static variables: $i_t$

# Simplifying the IS curve

- Notice that the model can be reduced to three equations by inserting the MP curve into the IS curve.

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}[i_t - \mathbb{E}_t \pi_{t+1} - r_t^n] \qquad (IS)$$

$$\downarrow \qquad \nwarrow i_t = \pi_t + r_t^n + \phi_\pi \pi_t + \phi_y \hat{y}_t \qquad (MP)$$

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left[\pi_t + r_t^n + \phi_\pi \pi_t + \phi_y \hat{y}_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right]$$

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left[(1 + \phi_\pi)\pi_t + \phi_y \hat{y}_t - \mathbb{E}_t \pi_{t+1}\right]$$

$$\frac{1}{\sigma}\mathbb{E}_t \pi_{t+1} + 1\mathbb{E}_t \hat{y}_{t+1} = \left(\frac{1 + \phi_\pi}{\sigma}\right)\pi_t + \left(\frac{\phi_y}{\sigma} + 1\right)\hat{y}_t$$

# 3 equations vs 3 unknowns

- The three equations

$$\frac{1}{\sigma}\mathbb{E}_t\pi_{t+1} + 1\mathbb{E}_t\hat{y}_{t+1} = \left(\frac{1+\phi_\pi}{\sigma}\right)\pi_t + \left(\frac{\phi_y}{\sigma} + 1\right)\hat{y}_t \tag{IS}$$

$$\mathbb{E}_t\pi_{t+1} = \pi_t - \kappa\hat{y}_t - s_t \tag{AS}$$

$$s_{t+1} = \rho_s s_t + \varepsilon^s_{t+1} \tag{Supply shock}$$

- The three unknowns
  - $\pi_t$ , $\hat{y}_t$ , $s_t$, for $t = 1, \ldots, n$

# Matrix representation

$$1 s_{t+1} + 0 \mathbb{E}_t \pi_{t+1} + 0 \mathbb{E}_t \hat{y}_{t+1} = \rho_s s_t + 0 \pi_t + 0 \hat{y}_t + 1 \varepsilon^s_{t+1}$$

$$0 s_{t+1} + \beta \mathbb{E}_t \pi_{t+1} + 0 \mathbb{E}_t \hat{y}_{t+1} = -1 s_t + 1 \pi_t - \kappa \hat{y}_t + 0 \varepsilon^\pi_{t+1}$$

$$0 s_{t+1} + \frac{1}{\sigma} \mathbb{E}_t \pi_{t+1} + 1 \mathbb{E}_t \hat{y}_{t+1} = 0 s_t + \left( \frac{1 + \phi_\pi}{\sigma} \right) \pi_t + \left( \frac{\phi_y}{\sigma} + 1 \right) \hat{y}_t + 0 \varepsilon^y_{t+1}$$

..

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 1/\sigma & 1 \end{bmatrix} \begin{bmatrix} s_{t+1} \\ \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \hat{y}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_s & 0 & 0 \\ -1 & 1 & -\kappa \\ 0 & \left( \frac{1+\phi_\pi}{\sigma} \right) & \left( \frac{\phi_y}{\sigma} + 1 \right) \end{bmatrix} \begin{bmatrix} s_t \\ \pi_t \\ \hat{y}_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon^s_{t+1} \\ \varepsilon^\pi_{t+1} \\ \varepsilon^y_{t+1} \end{bmatrix}$$

# The model is ready for the computer